Equivalent plastic strain vs tensor plastic strain

Introduction

In this article we want to clarify some doubts about plastic strain usage and correlation to stress.

Discussion

In OptiStruct 2 different plastic strain results are available, namely the tensor plastic strain and the equivalent plastic strain, that is a scalar quantity. Plastic strain output can be requested with the card STRAIN(PLASTIC)=YES. This request will create 2 separate outputs for the strain, total strain, and plastic strain, in addition to the equivalent plastic strain, where plastic refers to the **quota of the strain that cannot be recovered if the load is removed**.

Equivalent plastic strain takes into account the z-component of strain tensor for shells. On the other hand, strain tensor result does not output z-component of the tensor, which produces a difference in equivalent plastic strain and, for example, von Mises based on plastic strain tensor.

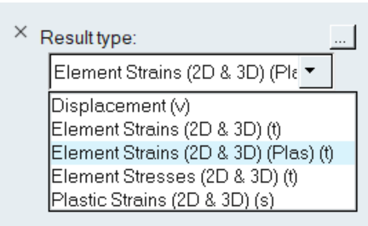


Figure 1: different strain results

We will show this. From a mathematical point of view, let e^{pl}_{ij} denote the plastic strain deviator and $\dot{e}^{pl}_{ij}$ its rate. Equivalent plastic strain is defined as:

$$\varepsilon_{ps} = \sqrt{\frac{2}{3}} \int_0^t \sqrt{\dot{e}^{pl}_{ij}\dot{e}^{pl}_{ij}}\, dt$$

It determines the length of trajectory in the 9-dimensional plastic strain space. It is assumed that the indices *i, j* range over values of 1,2,3 and the standard summation holds for the repeated indices.

Von Mises plastic strain is defined as:

$$\varepsilon_{Mises} = \sqrt{\frac{2}{3}} \sqrt{e^{pl}_{ij} e^{pl}_{ij}}$$

It is a measure of the shape plastic distortion of an infinitesimal element of the medium. In general, the 2 quantities are not the same,

$$\varepsilon_{ps} \neq \varepsilon_{Mises}$$

On one hand:

$$\dot{\varepsilon}_{ps} = \sqrt{\frac{2}{3}}\sqrt{\dot{e}^{pl}_{ij}\dot{e}^{pl}_{ij}}$$

On the other hand,

$$\dot{\varepsilon}_{Mises} = \sqrt{\frac{2}{3}}\frac{1}{2\sqrt{e^{pl}_{i'j'}e^{pl}_{i'j'}}}\left(2e^{pl}_{ij}\dot{e}^{pl}_{ij}\right) = \frac{2}{3\varepsilon_{Mises}}\left(e^{pl}_{ij}\dot{e}^{pl}_{ij}\right).$$

Clearly,

$$\sqrt{\frac{2}{3}}\sqrt{\dot{e}^{pl}_{ij}\dot{e}^{pl}_{ij}} \neq \frac{2}{3\varepsilon_{Mises}}\left(e^{pl}_{ij}\dot{e}^{pl}_{ij}\right)$$

Maximum equivalent plastic strain is reported in the last column of non-linear iterations in the .out file.

```
 Subcase          2  Load step: 1.2000E+00
-----------------------------------------------------------------------------------------------
                 Nonlinear Err. Measures   Gap and Contact Element Status MaxEquiv
Iter Avg. U     EUI       EPI       EWI     Open Closed Stick Slip Frozen PlasStrn
-----------------------------------------------------------------------------------------------
  1  3.84E-04 4.85E-02 1.01E-06 9.83E-06    0     0     0     0     0   7.79E-02
  2  3.84E-04 3.57E-05 2.26E-12 6.67E-15    0     0     0     0     0   7.79E-02
```

Test model

We will use the plate of Figure 2 to correlate stress and strain quantities. The model is subjected to a sequence of 3 load cases: Tension, Unloading and Shear deformation.

In the 1st lc the plate is pulled along x direction to deform it in a plastic way, in the 2nd lc the load is removed and the plate shows a spring back behavior, in 3rd lc the plate is subjected to a shear load. The 3 lc are in continuation.

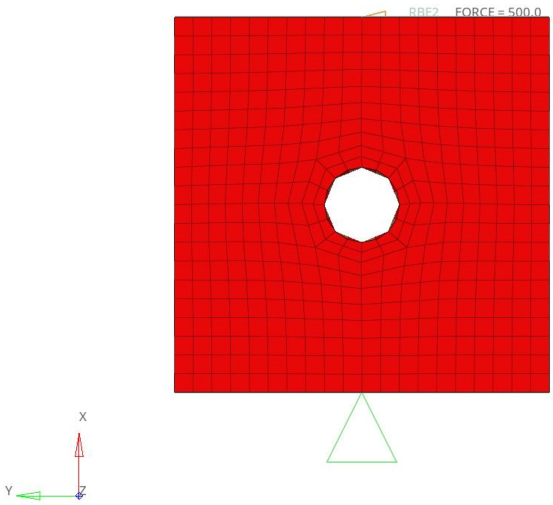


Figure 2: Test model

The material stress-strain curve is input with TYPSTRN =1, that means plastic strain is used on the x-axis and the curve is shown Figure 3. Yield point is at 359Mpa.

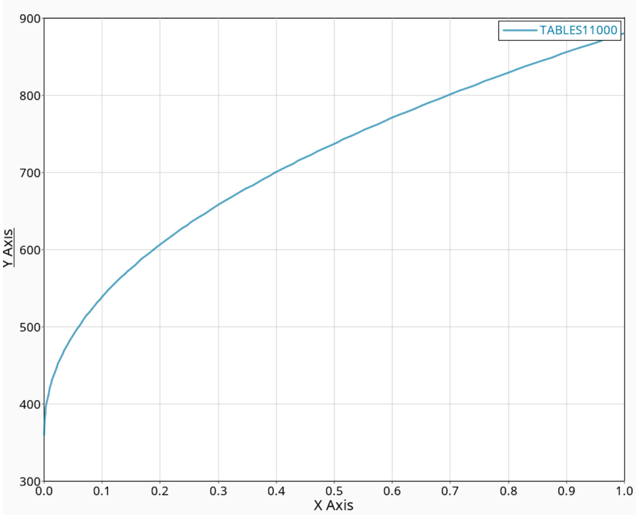


Figure 3: Stress-strain curve for the plate

Results for solid elements are shown in the next images for the 3 load cases.

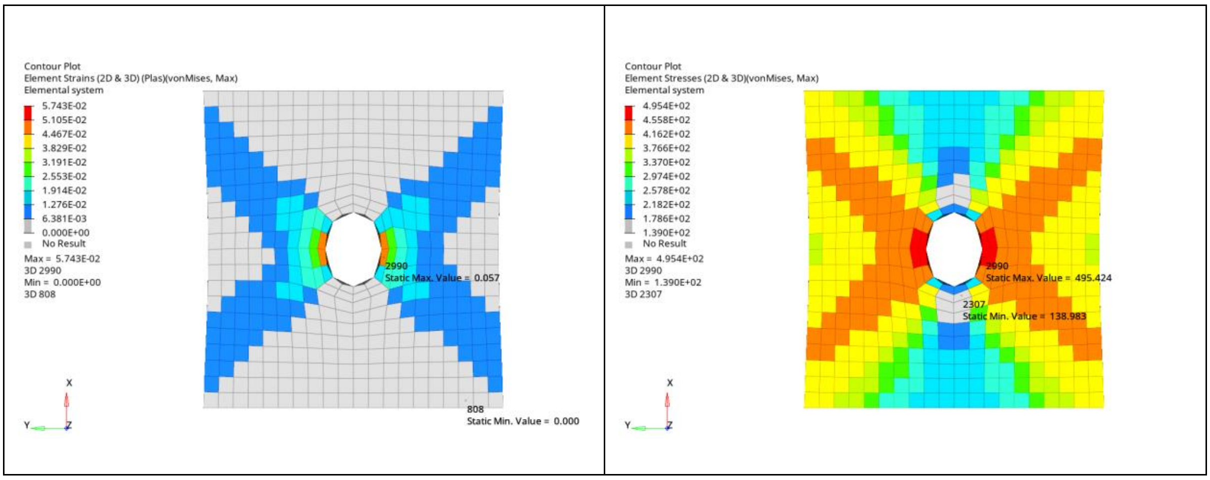


Table 1: VonM plastic strain (sx) and VonM stress (dx) - lc1

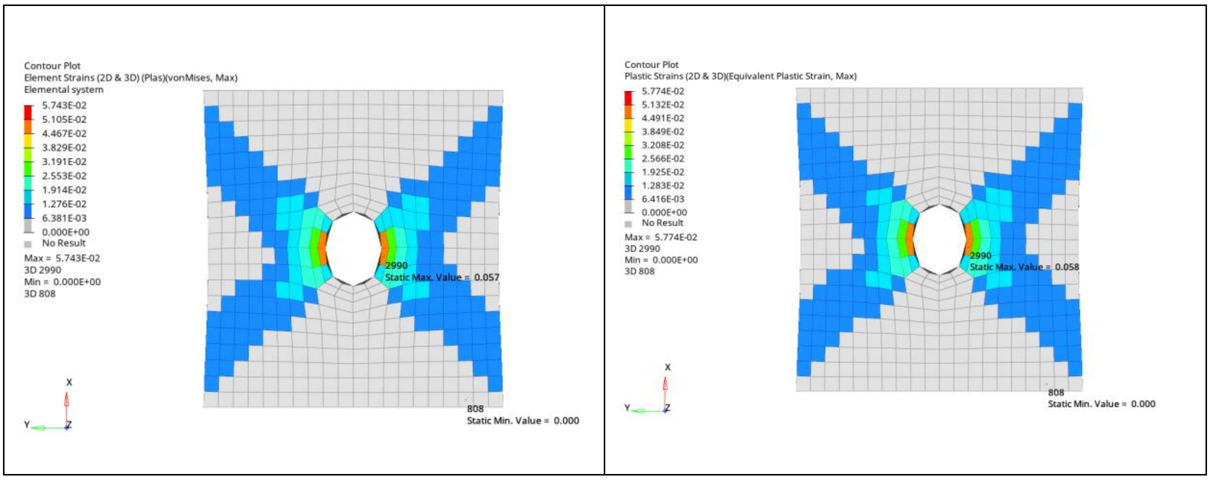


Table 2: Comparison of VonM plastic strain (sx) and Equivalent plastic strain (dx) for solids – lc1

Lc1 has a maximum voM plastic strain of 0.057 that corresponds to a VonM of 495Mpa on solid elements. The maximum equivalent plastic strain has a similar value of 0.058. During the lc2 the tension load is removed and the plate shows a spring back effect. You can observe that the maximum stress is reduced to 301Mpa below the yield limit of 359Mpa, but the plastic strain remains the same. This is correct because plastic deformation cannot be recovered.

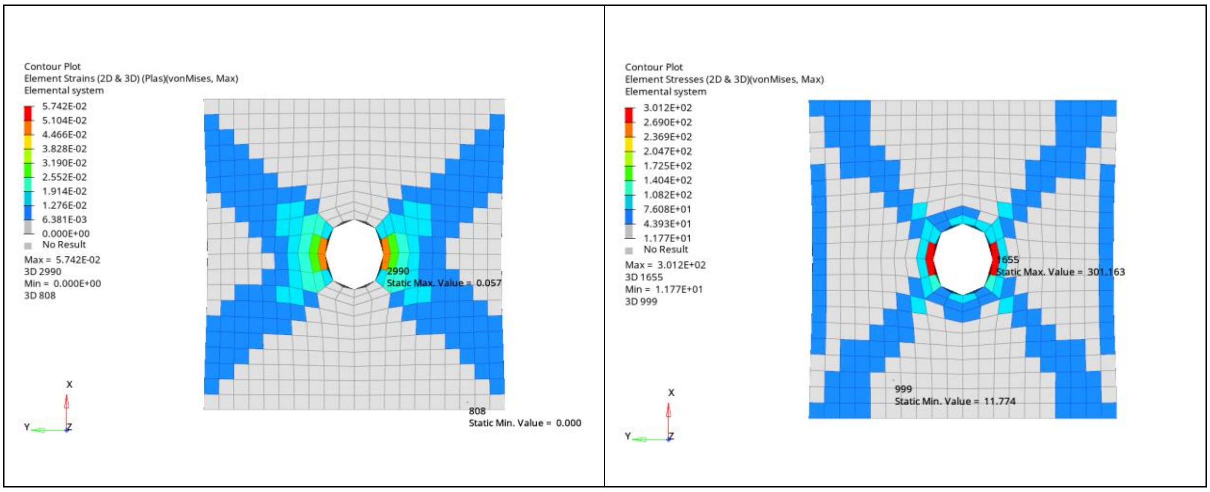


Table 3: VonM plastic strain (sx) and VonM stress (dx) – lc2

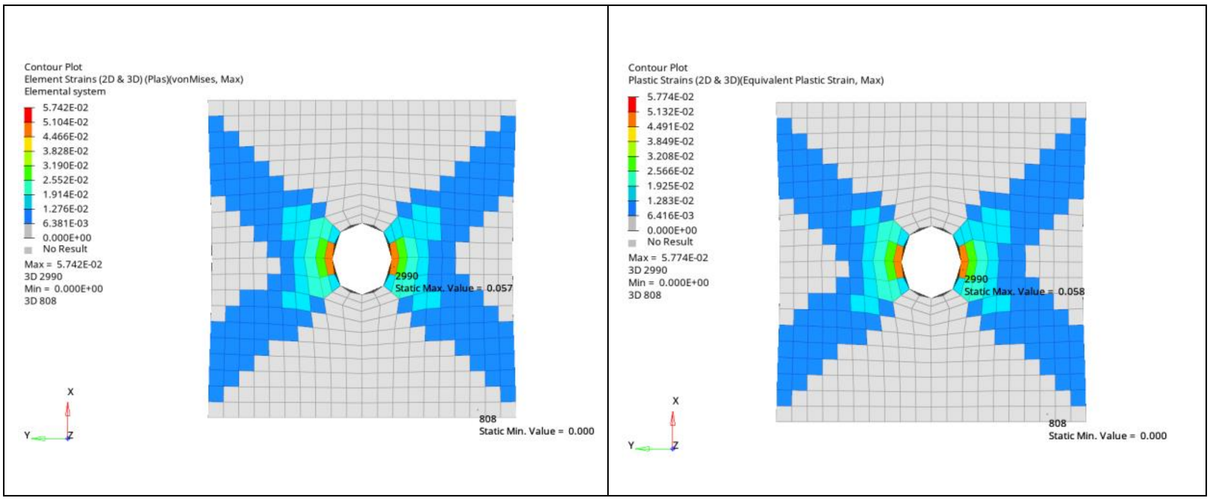


Table 4: Comparison of VonM plastic strain (sx) and Equivalent plastic strain (dx) for solids – lc2

In the 3rd lc a shear force is applied, the maximum stress reaches 374 just above the yield, and the maximum VonM plastic strain is almost unchanged.

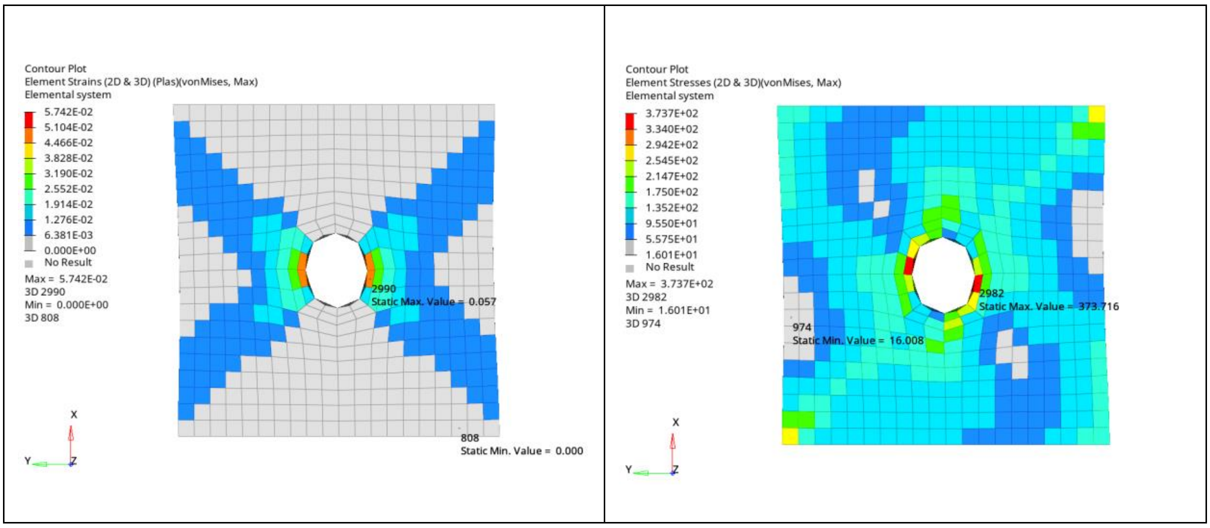


Table 5: VonM plastic strain (sx) and VonM stress (dx) – lc3

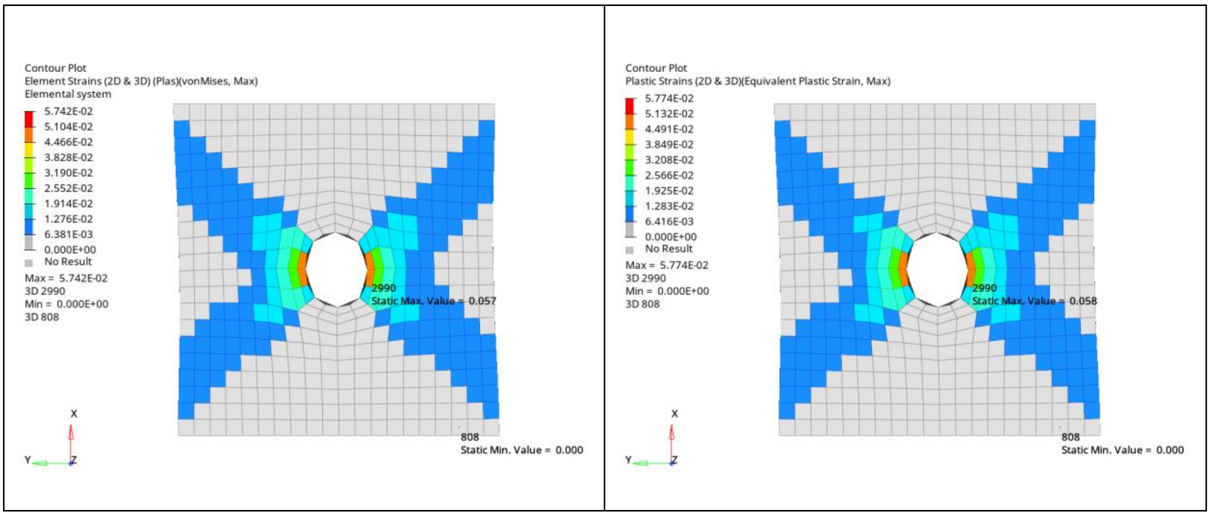


Table 6: Comparison of VonM plastic strain (sx) and Equivalent plastic strain (dx) for solids – lc3

For solid elements there is a negligible difference between VonM and equivalent plastic strain. This is not the case with shells for the reason described above.

The comparison is now shown on a thin shell layer wrapping the solid elements. In this case the difference between VonM and equivalent plastic strain is more important, 0.065 vs 0.078, and the value that better fits the stress strain curve of Figure 4 is the equivalent plastic strain, based on the interpolation of a computed stress of 519Mpa.

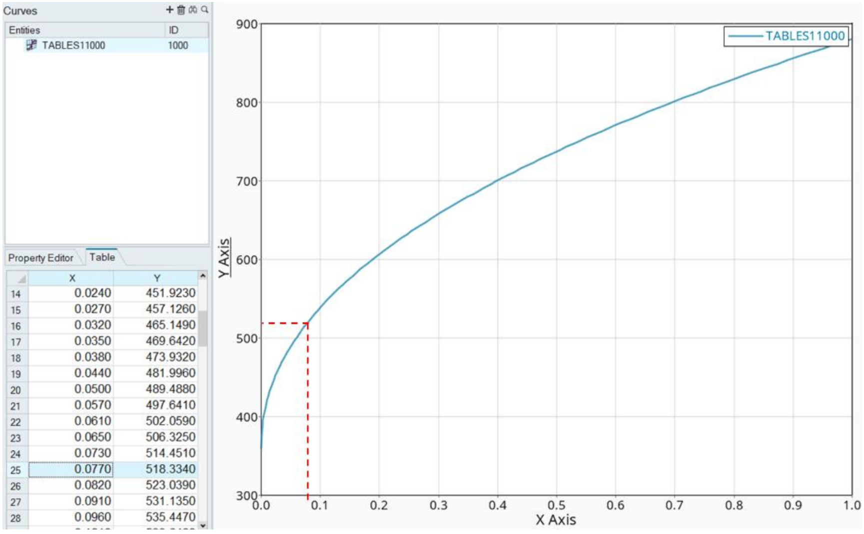
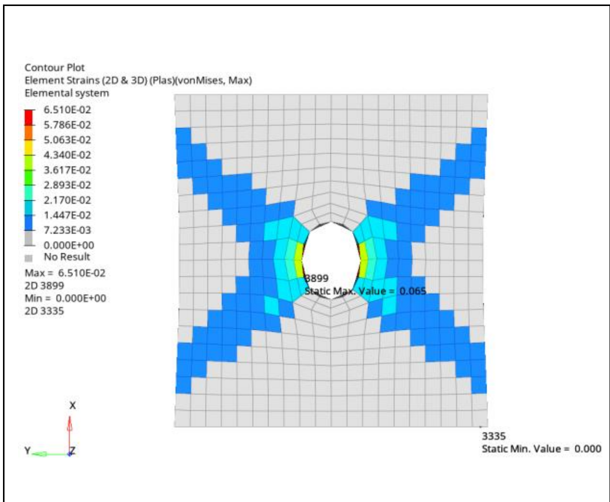


Figure 4: strain interpolation on shells



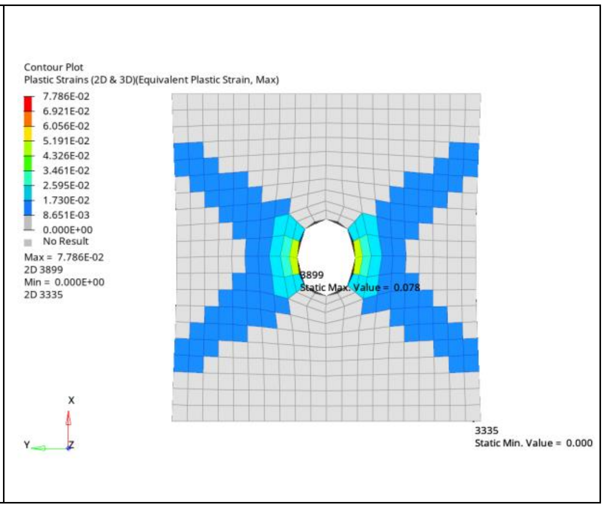
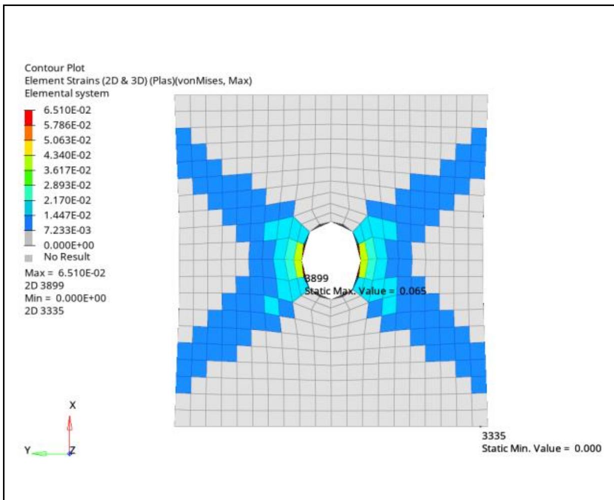


Table 7: Comparison of VonM plastic strain (sx) and Equivalent plastic strain (dx) for shells – lc1



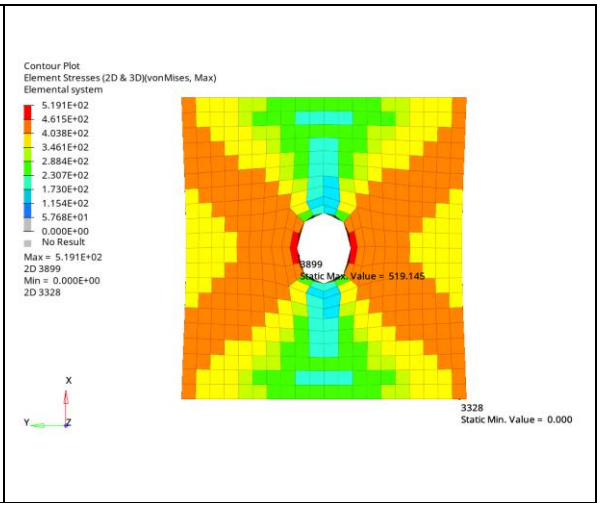


Table 8: VonM plastic strain (sx) and VonM stress (dx) for shells – lc1

Conclusion

I. Equivalent plastic strains are not computed in the same way as tensor plastic strains, this is evident for shell elements.

II. Plastic strains are not removed from the results during complex loading sequences with subcase continuations. That means there could not be correspondence between plastic strain and stress in specific situations.

III. For increased accuracy you can check results at Gauss points with the request STRAIN(CORNER, GAUSS, PLASTIC)=ALL.