```
dat = OpenWrite["results.dat", FormatType -> OutputForm,
  PageWidth -> 150]
datp = OpenWrite["res-p.dat", FormatType -> OutputForm,
  PageWidth -> 150]
dates = OpenWrite["res-es.dat", FormatType -> OutputForm,
  PageWidth -> 150]
temd = OpenWrite["tem.dat", FormatType -> OutputForm, PageWidth -> 150]

(* Input Data *)
Fstlayt = 0.6; Lstlayt = 1.6; Sndlayt = 0.3;
ls1 = 0.01; ls2 = 0.01; ls3 = 0.01;
r1 = 226.0;
ri = r1;
r2 = r1 + Fstlayt;
r3 = r2 + Sndlayt;
r4 = r3 + Lstlayt;
ro = r4;

K1 = 2.0;    K3 = 11.7;          (*watt/m^2c*)
K2[r_] := 1/(r2 - r3)*((K1 - K3)*r + (r2*K3 - r3*K1));
(*K1=2.0;*)
(*K2=K1;*)
(*K3=K1;*)
\[Alpha]1 = 9.5*10^-6; \[Alpha]3 = 11.1*10^-6;
\[Alpha]2[r_] :=
  1/(r2 - r3)*((\[Alpha]1 - \[Alpha]3)*
     r + (r2*\[Alpha]3 - r3*\[Alpha]1));
(*\[Alpha]1=1.1*10^-6;*)
(*\[Alpha]2=\[Alpha]1;*)
```

```
(*\[Alpha]3=\[Alpha]1;*)
T1 = 1100.0; T4 = 950.0; (*Degree Celcius*)
p1 = 10.0 ;  (*MPa*)
p4 = 0;
pesi = 0;
peso = 0;
init = 0. &;
(* Material Properties*)
\[Nu]1 = 0.2;  \[Nu]3 = 0.3;
\[Nu]2[r_] :=
  1/(r2 - r3)*((\[Nu]1 - \[Nu]3)*r + (r2*\[Nu]3 - r3*\[Nu]1));
(*\[Nu]1=0.25;*)
(*\[Nu]2=\[Nu]1;*)
(*\[Nu]3=\[Nu]1;*)
E1 = 100*10^3; E3 = 221*10^3;   (*MPa*)
E2[r_] := 1/(r2 - r3)*((E1 - E3)*r + (r2*E3 - r3*E1));
(*E1=100*10^6;*)
(*E2=E1;*)
(*E3=E1;*)
ttl = IntegerPart[2 + (r3 - r2)/ls2]
RT[1] = Log[r2/r1]/(2.0*Pi*K1);
(*RT2=Log[r3/r2]/(2.0*Pi*K2);*)
RT[ttl] = Log[r4/r3]/(2.0*Pi*K3);
For[i = 2, i < ttl - 1, i++,
  RT[i] =
    Log[(r2 + (i - 1)*ls2)/(
      r2 + (i - 2)*ls2)]/(2.0*Pi*K2[r2 + (i - 2)*ls2 + ls2/2]);
  ];
ct = Array[init, {ttl, ttl}];
```

```
MatrixForm[ct];
ct[[1, 1]] = 1;
ct[[1, ttl]] = RT[1];
ct[[ttl, ttl - 1]] = 1;
ct[[ttl, ttl]] = -RT[ttl];
For[i = 2, i < ttl, i++,
  ct[[i, i - 1]] = 1;
  ct[[i, i]] = -1;
  ];

For[i = 2, i < ttl, i++,
  ct[[i, ttl]] = -RT[i];
  ];
MatrixForm[ct]
bt = Array[init, ttl];
bt[[1]] = T1;
bt[[ttl]] = T4;
MatrixForm[bt]
xt = LinearSolve[ct, bt];
MatrixForm[xt]

(*a={{RT1+RT2,-RT1},{RT3,-(RT3+RT2)}};*)
(*b={T1*RT2,-T4*RT2};*)
(*inva=Inverse[a];*)
(*MatrixForm[inva];*)
(*sol=inva.b;*)
(*{T2,T3}=sol;*)
(*MatrixForm[sol];*)
```

```
(*c11=(T1-T2)/Log[r1/r2];c21=(T2*Log[r1]-T1*Log[r2])/Log[r1/r2];*)
(*c12=(T2-T3)/Log[r2/r3];c22=(T3*Log[r2]-T2*Log[r3])/Log[r2/r3];*)
(*c13=(T3-T4)/Log[r3/r4];c23=(T4*Log[r3]-T3*Log[r4])/Log[r3/r4];*)
(*t[r_]:=c11*Log[r]+c21/;r1\[LessEqual]r<r2;*)
t[r_] := c12*Log[r] + c22 /; r2 <= r < r3;
t[r_] := c13*Log[r] + c23 /; r3 <= r <= r4;

(*Stress Calculation*)
c1 = r1/r2; c2 = r2/r3; c3 = r3/r4;
\[Delta]10 = (2*c1*(1 - \[Nu]1^2))/(E1*(1 - c1^2));
\[Delta]11 = -((1 + \[Nu]1)/(E1*(1 - c1^2)))*(1 + c1^2 - 2*\[Nu]1) - (
   1 + \[Nu]2)/(E2*(1 - c2^2))*(1 + c2^2 - 2*\[Nu]2*c2^2);
\[Delta]12 = (2*c2*(1 - \[Nu]2^2) )/(E2*(1 - c2^2));
\[Delta]21 = (2*c2*(1 - \[Nu]2^2))/(E2*(1 - c2^2));
\[Delta]22 = -((1 + \[Nu]2)/(E2*(1 - c2^2)))*(1 + c2^2 - 2*\[Nu]2) - (
   1 + \[Nu]3)/(E3*(1 - c3^2))*(1 + c3^2 - 2*\[Nu]3*c3^2);
\[Delta]23 = (2*c3*(1 - \[Nu]3^2))/(E3*(1 - c3^2));

(* A Matrix for pressure*)
ap = {{\[Delta]11*r2, \[Delta]12*r3}, {\[Delta]21*r2, \[Delta]22*r3}};
MatrixForm[ap];

(* B Matrix*)
bp = {-\[Delta]10*r1*p1, -\[Delta]23*r4*p4};
MatrixForm[bp];
solp = Inverse[ap].bp;
{p2, p3} = solp;
MatrixForm[solp];
```

```
\[Sigma]rp[
  r_] := ((c1)^2*p1)/(1 - (c1)^2)*(1 - (r2)^2/r^2) -
   p2/(1 - (c1)^2)*(1 - (c1)^2*(r2)^2/r^2) /; r1 <= r < r2;
\[Sigma]rp[
  r_] := ((c2)^2*p2)/(1 - (c2)^2)*(1 - (r3)^2/r^2) -
   p3/(1 - (c2)^2)*(1 - (c2)^2*(r3)^2/r^2) /; r2 <= r < r3;
\[Sigma]rp[
  r_] := ((c3)^2*p3)/(1 - (c3)^2)*(1 - (r4)^2/r^2) -
   p4/(1 - (c3)^2)*(1 - (c3)^2*(r4)^2/r^2) /; r3 <= r <= r4;
\[Sigma]\[Theta]p[
  r_] := ((c1)^2*p1)/(1 - (c1)^2)*(1 + (r2)^2/r^2) -
   p2/(1 - (c1)^2)*(1 + (c1)^2*(r2)^2/r^2) /; r1 <= r < r2;
\[Sigma]\[Theta]p[
  r_] := ((c2)^2*p2)/(1 - (c2)^2)*(1 + (r3)^2/r^2) -
   p3/(1 - (c2)^2)*(1 + (c2)^2*(r3)^2/r^2) /; r2 <= r < r3;
\[Sigma]\[Theta]p[
  r_] := ((c3)^2*p3)/(1 - (c3)^2)*(1 + (r4)^2/r^2) -
   p4/(1 - (c3)^2)*(1 + (c3)^2*(r4)^2/r^2) /; r3 <= r <= r4;

urp[r_] := ((1 + \[Nu]1)*(1 - 2*\[Nu]1)*r2)/(
   E1*(1 - (c1)^2))*((c1)^2*p1*(r/r2 + r2/(r*(1 - 2*\[Nu]1))) -
     p2*(r/r2 + (r2*(c1)^2)/(r*(1 - 2*\[Nu]1)))) /; r1 <= r < r2;
urp[r_] := ((1 + \[Nu]2)*(1 - 2*\[Nu]2)*r3)/(
   E2*(1 - (c2)^2))*((c2)^2*p2*(r/r3 + r3/(r*(1 - 2*\[Nu]2))) -
     p3*(r/r3 + (r3*(c2)^2)/(r*(1 - 2*\[Nu]2)))) /; r2 <= r < r3;
urp[r_] := ((1 + \[Nu]3)*(1 - 2*\[Nu]3)*r4)/(
   E3*(1 - (c3)^2))*((c3)^2*p3*(r/r4 + r4/(r*(1 - 2*\[Nu]3))) -
     p4*(r/r4 + (r4*(c3)^2)/(r*(1 - 2*\[Nu]3)))) /; r3 <= r <= r4;
\[Sigma]\[Theta]p1[r_] := ((c1)^2*p1)/(1 - (c1)^2)*(1 + (r2)^2/r^2) -
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   p2/(1 - (c1)^2)*(1 + (c1)^2*(r2)^2/r^2);
\[Sigma]\[Theta]p2[r_] := ((c2)^2*p2)/(1 - (c2)^2)*(1 + (r3)^2/r^2) -
   p3/(1 - (c2)^2)*(1 + (c2)^2*(r3)^2/r^2);
\[Sigma]\[Theta]p3[r_] := ((c3)^2*p3)/(1 - (c3)^2)*(1 + (r4)^2/r^2) -
   p4/(1 - (c3)^2)*(1 + (c3)^2*(r4)^2/r^2);

(* A Matrix for eigenstrain*)

ln1 = (r2 - r1)/ls1;
ln2 = (r3 - r2)/ls2;
ln3 = (r4 - r3)/ls3;
tnl = IntegerPart[ln1 + ln2 + ln3];
el[x_] := Which[r1 <= x < r2, E1, r2 <= x < r3, E2, r3 <= x <= r4, E3];
kl[x_] := Which[r1 <= x < r2, K1, r2 <= x < r3, K2, r3 <= x <= r4, K3];
nul[x_] :=
  Which[r1 <= x < r2, \[Nu]1, r2 <= x < r3, \[Nu]2,
   r3 <= x <= r4, \[Nu]3];
alfl[x_] :=
  Which[r1 <= x < r2, \[Alpha]1, r2 <= x < r3, \[Alpha]2,
   r3 <= x <= r4, \[Alpha]3];
For[i = 0, i < tnl + 1, i++,
  rl[i] = r1 + i*ls1;
  ];
For[i = 1, i < tnl + 1, i++,
  c[i] = rl[i - 1]/rl[i];
  ];
For[i = 1, i < tnl, i++,
  x = rl[i] - ls1/2;
  y = rl[i] + ls1/2;
```

```
 \[Delta][i, i - 1] = (2*c[i]*(1 - (nul[x])^2))/(
  el[x]*(1 - (c[i])^2));
 \[Delta][i,
   i] = -((1 + nul[x])/(
     el[x]*(1 - (c[i])^2)))*(1 + (c[i])^2 - 2*nul[x]) - (
    1 + nul[y])/(
    el[y]*(1 - (c[i + 1])^2))*(1 + (c[i + 1])^2 -
      2*nul[y]*(c[i + 1])^2);
 \[Delta][i, i + 1] = (2*c[i + 1]*(1 - (nul[y])^2))/(
  el[y]*(1 - (c[i + 1])^2));
 ];
aes = Array[init, {tnl - 1, tnl - 1}];
bes = Array[init, tnl - 1];
aes[[1, 1]] = \[Delta][1, 1]*rl[1];
aes[[1, 2]] = \[Delta][1, 2]*rl[2];
aes[[tnl - 1, tnl - 1]] = \[Delta][tnl - 1, tnl - 1]*rl[tnl - 1];
aes[[tnl - 1, tnl - 2]] = \[Delta][tnl - 1, tnl - 2]*rl[tnl - 2];
x = rl[1] - ls1/2;
y = rl[1] + ls1/2;
bes[[1]] =
 rl[1]*((1 + nul[y])*t[y]*alfl[y] - (1 + nul[x])*t[x]*
     alfl[x]) - \[Delta][1, 0]*rl[0]*pesi;
x = rl[tnl - 1] - ls1/2;
y = rl[tnl - 1] + ls1/2;
bes[[tnl - 1]] =
 rl[tnl - 1]*((1 + nul[y])*t[y]*alfl[y] - (1 + nul[x])*t[x]*
     alfl[x]) - \[Delta][tnl - 1, tnl]*rl[tnl]*peso;

For[i = 2, i < tnl - 1, i++,
```

```
  aes[[i, i - 1]] = \[Delta][i, i - 1] rl[i - 1];
  aes[[i, i]] = \[Delta][i, i]*rl[i];
  aes[[i, i + 1]] = \[Delta][i, i + 1]*rl[i + 1];
  x = rl[i] - ls1/2;
  y = rl[i] + ls1/2;
  bes[[i]] =
   rl[i]*((1 + nul[y])*t[y]*alfl[y] - (1 + nul[x])*t[x]*alfl[x]);
  ];
soles = Inverse[aes].bes;
pes = Array[init, tnl];
pesf = Array[init, tnl + 1];
pes = Insert[soles, pesi, {1}];
pesf = Insert[pes, peso, {-1}];
MatrixForm[soles];
MatrixForm[pes];
MatrixForm[pesf];


\[Sigma]r[
   r_] := ((c[tnl])^2*pesf[[tnl]])/(
     1 - (c[tnl])^2)*(1 - (rl[tnl])^2/r^2) -
    pesf[[tnl + 1]]/(
     1 - (c[tnl])^2)*(1 - (c[tnl])^2*(rl[tnl])^2/r^2) /; r4 <= r;
\[Sigma]\[Theta][
   r_] := ((c[tnl])^2*pesf[[tnl]])/(
     1 - (c[tnl])^2)*(1 + (rl[tnl])^2/r^2) -
    pesf[[tnl + 1]]/(
     1 - (c[tnl])^2)*(1 + (c[tnl])^2*(rl[tnl])^2/r^2) /; r4 <= r;
ur[r_] := ((1 + nul[r])*(1 - 2*nul[r])*rl[tnl])/(
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    el[r]*(1 - (c[tnl])^2))*((c[tnl])^2*
       pesf[[tnl]]*(r/rl[tnl] + rl[tnl]/(r*(1 - 2*nul[r]))) -
      pesf[[tnl + 1]]*(r/rl[tnl] + (rl[tnl]*(c[tnl])^2)/(
        r*(1 - 2*nul[r])))) + (1 + nul[r])*r*t[r]*alfl[r] /; r4 <= r;

\[Sigma]r[
  r_] := ((c[IntegerPart[(r - r1)/ls1] + 1])^2*
      pesf[[IntegerPart[(r - r1)/ls1] + 1]])/(1 - (c[
       IntegerPart[(r - r1)/ls1] + 1])^2)*(1 - (rl[
       IntegerPart[(r - r1)/ls1] + 1])^2/r^2) -
   pesf[[IntegerPart[(r - r1)/ls1] + 2]]/(
    1 - (c[IntegerPart[(r - r1)/ls1] +
       1])^2)*(1 - (c[IntegerPart[(r - r1)/ls1] + 1])^2*(rl[
       IntegerPart[(r - r1)/ls1] + 1])^2/r^2) /; r1 <= r < r4;
\[Sigma]\[Theta][
  r_] := ((c[IntegerPart[(r - r1)/ls1] + 1])^2*
      pesf[[IntegerPart[(r - r1)/ls1] + 1]])/(1 - (c[
       IntegerPart[(r - r1)/ls1] + 1])^2)*(1 + (rl[
       IntegerPart[(r - r1)/ls1] + 1])^2/r^2) -
   pesf[[IntegerPart[(r - r1)/ls1] + 2]]/(
    1 - (c[IntegerPart[(r - r1)/ls1] +
       1])^2)*(1 + (c[IntegerPart[(r - r1)/ls1] + 1])^2*(rl[
       IntegerPart[(r - r1)/ls1] + 1])^2/r^2) /; r1 <= r < r4;
ur[r_] := ((1 + nul[r])*(1 - 2*nul[r])*(rl[
      IntegerPart[(r - r1)/ls1] + 1]))/(
    el[r]*(1 - (c[IntegerPart[(r - r1)/ls1] + 1])^2))*((c[
       IntegerPart[(r - r1)/ls1] + 1])^2*
      pesf[[IntegerPart[(r - r1)/ls1] + 1]]*(r/
       rl[IntegerPart[(r - r1)/ls1] + 1] +
```

```
        rl[IntegerPart[(r - r1)/ls1] + 1]/(r*(1 - 2*nul[r])) -
      pesf[[IntegerPart[(r - r1)/ls1] + 2]]*(r/
        rl[IntegerPart[(r - r1)/ls1] +
          1] + ((rl[IntegerPart[(r - r1)/ls1] + 1])*(c[
            IntegerPart[(r - r1)/ls1] + 1])^2)/(r*(1 -
            2*nul[r]))))) + (1 + nul[r])*r*t[r]*alfl[r] /;
  r1 <= r <= r4;
Plot[urp[r], {r, r1, r4}];
Plot[\[Sigma]rp[r], {r, r1, r4}];
Plot[\[Sigma]\[Theta]p[r], {r, r1, r4}];
Plot[ur[r] + urp[r], {r, r1, r4}];
Plot[\[Sigma]r[r] + \[Sigma]rp[r], {r, r1, r4}];
Plot[\[Sigma]\[Theta][r] + \[Sigma]\[Theta]p[r], {r, r1, r4}];

Write[dat, "t1=", T1, "   ", "t4=", T4, "  ", "p1=", p1, "  ", "p4=",
  p4];
Write[dat, "r", "   ", "rnd", "   ", "ur", "   ", "sr", "   ", "sthe"];
Write[dat, r1, "  ", (r1 - r1)/(r4 - r1), "  ",
  CForm[(ur[r1] + urp[r1])], "  ",
  CForm[(\[Sigma]r[r1] + \[Sigma]rp[r1])], "  ",
  CForm[(\[Sigma]\[Theta][r1] + \[Sigma]\[Theta]p[r1])]];
For[r = r1 + ls1/2, r < r4, r += ls1,
  rnd = (r - r1)/(r4 - r1);
  \[Sigma]rf = (\[Sigma]r[r] + \[Sigma]rp[r]);
  \[Sigma]\[Theta]f = (\[Sigma]\[Theta][r] + \[Sigma]\[Theta]p[r]);
  urf = (ur[r] + urp[r]);
  Write[dat, r, "      ", rnd, "  ", CForm[urf], "        ",
   CForm[\[Sigma]rf], "      ", CForm[\[Sigma]\[Theta]f]];
  ];
```

```
Write[dat, r4, "  ", (r4 - r1)/(r4 - r1), "  ",
  CForm[(ur[r4] + urp[r4])], "  ",
  CForm[(\[Sigma]r[r4] + \[Sigma]rp[r4])], "  ",
  CForm[(\[Sigma]\[Theta][r4] + \[Sigma]\[Theta]p[r4])]];
Close[dat]

Write[datp, "t1=", T1, "   ", "t4=", T4, "  ", "p1=", p1, "  ", "p4=",
   p4];
Write[datp, "r", "   ", "rnd", "  ", "urp", "   ", "srp", "   ",
  "sthep"];
Write[datp, r1, "  ", (r1 - r1)/(r4 - r1), "  ", CForm[urp[r1]], "  ",
   CForm[\[Sigma]rp[r1]], "  ", CForm[\[Sigma]\[Theta]p[r1]]];
For[r = r1 + ls1/2, r < r4, r += ls1,
  rnd = (r - r1)/(r4 - r1);
  \[Sigma]rpp = \[Sigma]rp[r];
  \[Sigma]\[Theta]pp = \[Sigma]\[Theta]p[r];
  urpp = urp[r];
  Write[datp, r, "      ", rnd, "  ", CForm[urpp], "       ",
   CForm[\[Sigma]rpp], "      ", CForm[\[Sigma]\[Theta]pp]];
  ];
Write[datp, r4, "  ", (r4 - r1)/(r4 - r1), "  ", CForm[urp[r4]], "  ",
   CForm[\[Sigma]rp[r4]], "  ", CForm[\[Sigma]\[Theta]p[r4]]];
Close[datp]

Write[dates, "t1=", T1, "   ", "t4=", T4, "  ", "p1=", p1, "  ",
  "p4=", p4];
Write[dates, "r", "   ", "rnd", "  ", "ures", "   ", "sres", "   ",
  "sthees"];
Write[dates, r1, "  ", (r1 - r1)/(r4 - r1), "  ", CForm[ur[r1]], "  ",
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```
  CForm[\[Sigma]r[r1]], "  ", CForm[\[Sigma]\[Theta][r1]]];
For[r = r1 + ls1/2, r < r4, r += ls1,
 rnd = (r - r1)/(r4 - r1);
 \[Sigma]res = \[Sigma]r[r];
 \[Sigma]\[Theta]es = \[Sigma]\[Theta][r];
 ures = ur[r];
 Write[dates, r, "      ", rnd, "  ", CForm[ures], "        ",
  CForm[\[Sigma]res], "      ", CForm[\[Sigma]\[Theta]es]];
 ];
Write[dates, r4, "  ", (r4 - r1)/(r4 - r1), "  ", CForm[ur[r4]], "  ",
  CForm[\[Sigma]r[r4]], "  ", CForm[\[Sigma]\[Theta][r4]]];
Close[dates]

Write[temd, "t1=", T1, "   ", "t4=", T4, "  ", "p1=", p1, "  ", "p4=",
  p4];
Write[temd, "r", "   ", "rnd", "  ", "t"];
For[r = r1, r < r4 + ls1, r += ls1,
 rnd = (r - r1)/(r4 - r1);
 td = t[r];
 Write[temd, r, "      ", rnd, "  ", CForm[td]];
 ];
Close[temd]
\[Sigma]r[r4];
ur[226.6]
ur[226.9]
ur[226.55]
```