AC analysis of DC-DC Boost Converter

Overview

This example demonstrates the assessment of the loop stability of a closed-loop DC-DC boost converter using Multi-tone Periodic AC (MPAC) analysis in SaberRD.
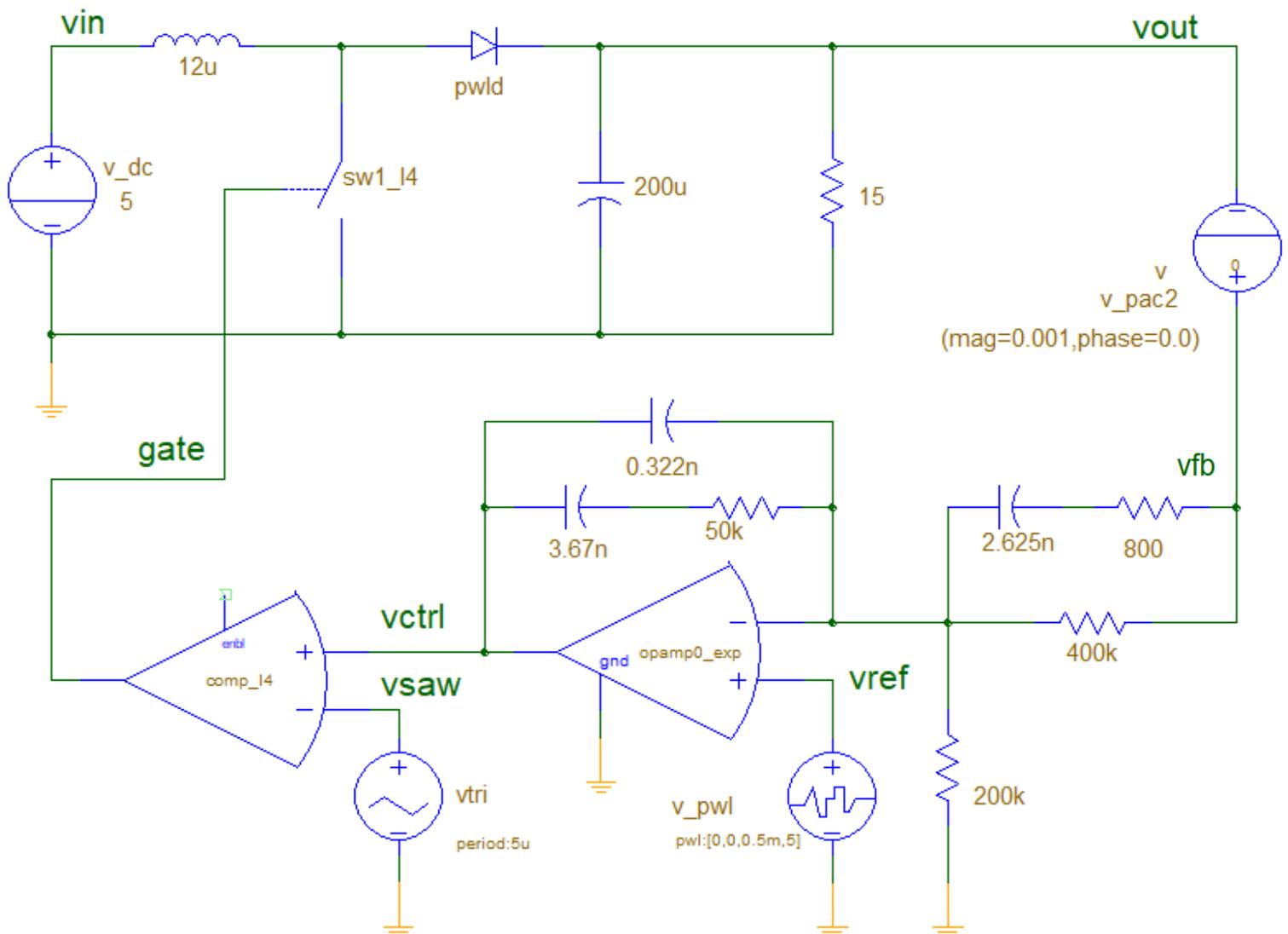

Figure 1: Closed-loop *DC-DC boost converter*

The voltage feedback loop is designed and tuned to ensure stable output voltage using frequency domain analysis. A type 3 compensator is used for closed-loop control.

Introduction

The DC-DC boost converter is the most commonly used topology among non-isolated power converter topologies. It is easy to control the input current from the source in power factor control applications and to control the DC output voltage to the desired voltage level. In a boost converter, as the name implies, the DC output voltage is greater than the DC input voltage.

This design example provides a method to perform frequency domain analysis of a closed-loop DC-DC boost converter using the Multi-tone PAC (MPAC) analysis in SaberRD. The MPAC analysis is used to produce Bode plots of switching and periodic systems. Transfer functions and Bode plots are useful to assess the stability of closed-loop circuits, using criteria like gain and phase margins, or the Middlebrook criterion.

Transfer Functions and Frequency Analysis

Block diagram and its transfer function

The block diagram of a typical closed-loop switching power supply is shown in Figure 2.
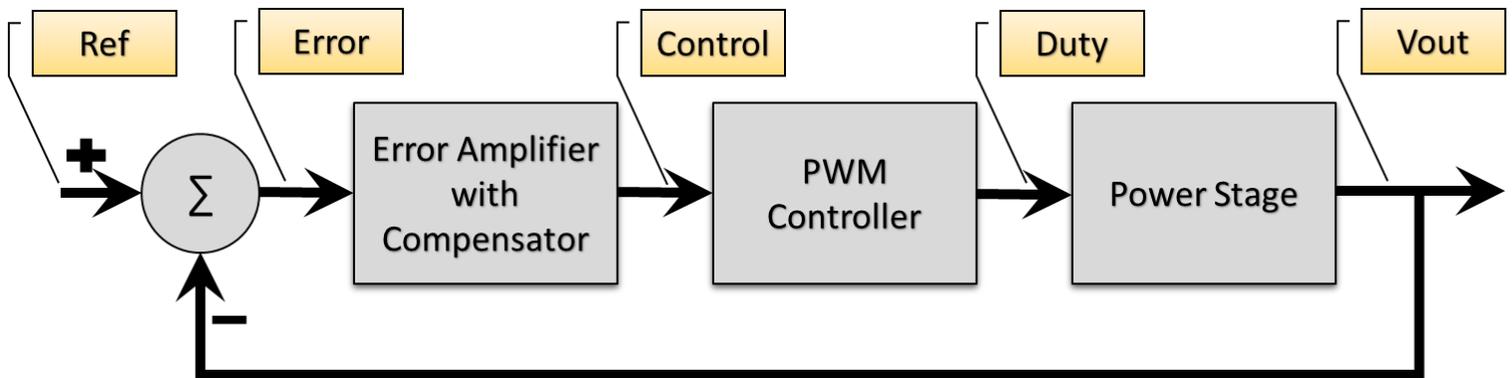

Figure 2: *Block diagram of a typical closed-loop switching power supply*

Individual blocks are represented by their transfer functions. The significance of each block is described below:

1. Power stage

This block represents the input source, switching devices, and output filters. The amplitude of the duty input decides the duty cycle at which the power stage operates. The transfer function for this block is normally represented as follows:

$$T_P(s) = \frac{Vout(s)}{Duty(s)}$$

2. Pulse Width Modulation (PWM) controller

Different power supplies require different levels of control schemes such as constant voltage control PWM, Sinusoidal PWM, Space Vector PWM, and so on. The transfer function for the PWM controller block is represented as follows:

$$Tm(s) = \frac{Duty(s)}{Control(s)}$$

3. Error amplifier and compensator

This block takes as input, the difference between the desired reference signal and the output voltage feedback signal. The output of this block is a control signal that is fed to the PWM controller to synthesize the desired duty cycle. This block is represented by $Tc(s)$, the transfer function of the compensated error amplifier which is used to close the loop.

$$Tc(s) = \frac{Control(s)}{Error(s)}$$

To ensure the stability of the closed-loop system, the following conditions are validated on the overall open-loop Bode plot:

1. The gain at lower frequencies should be higher.
2. Gain crossover frequency is as high as possible, but less than half of the switching frequency.
3. A phase margin of 45^0 to 60^0.

The overall open-loop Bode plot is the Bode plot of the transfer function given below

$$T_{OL}(s) = T_P(s) \times Tm(s) \times Tc(s)$$

The overall open-loop transfer function is nothing but the series connection of the three subsystems in the closed-loop system, described above. To assess stability, one of the following methods can be used:

1. The individual Bode plots can be obtained and then multiplied to get the system response.
2. Use the method described in the next section where the feedback path is broken, and the perturbation is introduced at the feedback node.

Transfer Function of the design

The Bode plot of the overall open-loop transfer function which aids in assessing stability is commonly measured by breaking the feedback loop and inserting the perturbation source between the output and the feedback node. The ratio of the frequency response of this feedback signal where the perturbation source is connected and that of the output gives the Bode plot of the overall open-loop transfer function. The paragraphs that follow, describe the process.

Let us examine the DC-DC boost converter schematic and configure it to get the overall open-loop transfer function. The schematic shown in Figure 1 is used in Figure 3 to represent the various blocks and the transfer functions in the closed-loop system discussed in Figure 2. As the reference (Ref) input is a constant DC input, its contribution to the frequency domain is zero. It can also be seen that the output voltage is not connected directly to the compensator. A perturbation source is added between the output node and the compensator input. So, the transfer function of this configuration is given by the equation below.

$$T_{OL}(s) = \frac{Vout(s)}{Feedback(s)} = \frac{Vout(s)}{Control(s)} \times \frac{Control(s)}{Feedback(s)} = [T_P(s) \times Tm(s)] \times Tc(s)$$

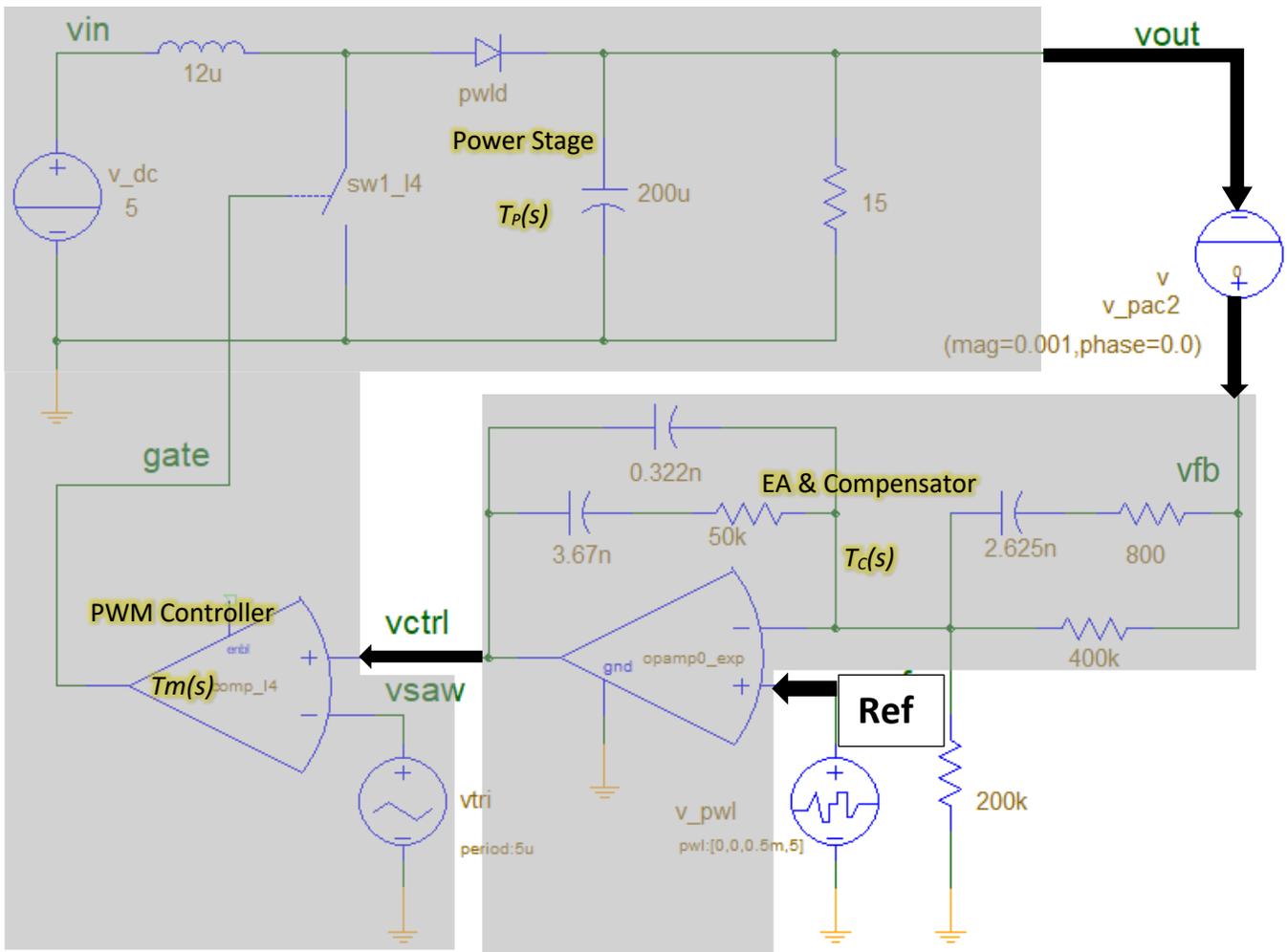So, Figure 3 can be represented in the form of a block diagram, as shown in Figure 4 to derive the transfer function.

Figure 3: *DC-DC boost converter design representing blocks and transfer functions*
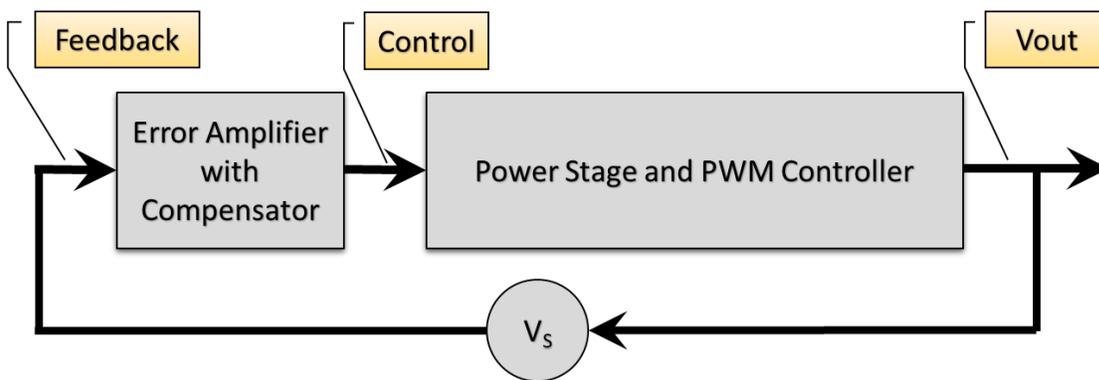

Figure 4: *Block diagram of the converter in simulation*

This establishes that by breaking the feedback loop and inserting the perturbation source, as shown in Figure 4, we can obtain the overall open-loop transfer function. Performing PAC analysis on this system will help to plot the Bode plot of the overall open-loop transfer function and assess its stability.

After performing PAC analysis, to get the Bode plot of the overall open-loop transfer function, it is required that the voltage at the input node of the perturbation source (for example, `vfb`) is divided by the same at the output node (for example, `vout`).

Performing MPAC Analysis in SaberRD

Multi-tone PAC Analysis

Traditionally, transfer functions of switching circuits are obtained by manually deriving average / non-switching models and linearizing them around an operating point. This tedious and error-prone task is eliminated by the PAC analysis, which produces Bode plots for switching circuits as-is. SaberRD provides an accurate and fast method to perform PAC analysis.

The PAC analysis is used to determine the frequency response of a switching system, which has a periodic steady state. A multiple frequency sinusoidal perturbation is injected at the input node of the system, and the outputs are measured to plot the Bode plots of the transfer functions. This way, the design that is used for time-domain simulation can be used without any changes for obtaining the Bode plots. In SaberRD, Multi-tone PAC analysis is supported.

In an MPAC simulation, a single transient analysis is performed to determine the frequency response of the system in a given operating condition. A general-purpose voltage source, available in the generic library, applies sinusoidal perturbation of different frequencies in the specified frequency range. The result of this simulation is the frequency response of the different signals selected as outputs. Calculating the ratio of appropriate signals gives the Bode plot of the required transfer function.

Boost Converter Design Considerations

The design specifications and the design equations for the power stage of the DC-DC boost converter are given in Table 1 and Table 2 respectively.

Table 1: *Design specifications*

Specification	Rating
Input voltage (V_{IN})	5 V +/-10 %
Output voltage (V_{OUT})	15 V
Output power (P_{OUT})	15 W
Switching frequency (F_{SW})	200 kHz
Inductor current ripple (ΔI_L)	40 %
Output voltage ripple (ΔV_O)	1 %
Load regulation (0.5 A to 1 A)	5 %

Table 2: *Design equations*

Parameter	Formula	Calculation
Maximum duty ratio (In CCM)	$D_{MAX} = 1 - \dfrac{V_{IN(MIN)}}{V_{OUT}}$	$D_{MAX} = 1 - \dfrac{4.5}{15} = 0.7$
Average inductor current	$I_{L(Avg)} = \dfrac{P_{OUT}}{V_{IN(MIN)}}$	$I_{L(Avg)} = \dfrac{15}{4.5} = 3.3 \ A$
Inductor current ripple	$\Delta I_L = I_{L(Avg)} \times 0.40$	$\Delta I_L = 1.32 \ A$
Boost inductance	$L = V_{IN(MIN)} \dfrac{(D_{MAX}/F_{SW})}{\Delta I_L}$	$L = \dfrac{4.5 \times \left(\frac{0.7}{200k}\right)}{1.32} = 11.9 \ \mu H$

Output filter capacitance (Assuming ESR = 0)	$C_{out} = I_{OUT} \dfrac{(D_{MAX}/F_{SW})}{\Delta V_O}$	$C_{out} = 1 \times \dfrac{\left(\dfrac{0.7}{200k}\right)}{0.15} = 23.3\ \mu F$

Note: A 200 µF capacitor was selected against the calculated value of 23.3 µF to improve the stability in closed-loop operation.

For the closed-loop control, a type 3 compensator is selected, and the output voltage is taken as feedback for the control. The compensator, shown in Figure 5, is designed with the frequencies mentioned in Table 3.
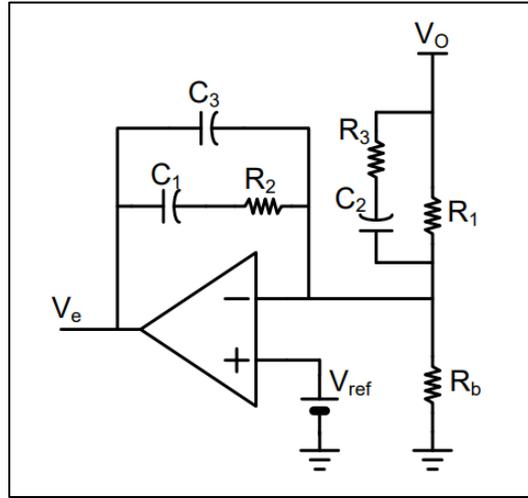
Figure 5: *Type 3 compensator*

The compensator has one pole at origin (F_{P0}), two zeros placed (F_{Z1} And F_{Z2}) before the gain crossover frequency (F_0), and two poles (F_{P1} and F_{P2}) after the gain crossover frequency.

Table 3: *Frequency parameters of type 3 compensator*

Parameter	Formula	Calculation
F (double poles)	*From Open-loop Bode plot*	900 Hz
F_{ESR}	$\dfrac{1}{2\pi \times ESR \times Co}$	$\dfrac{1}{2 \times 3.141 \times 10m \times 200u} = 79.6\ kHz$
F_0	$\dfrac{1}{40} \times Fsw$	5 kHz
F_{P0}	*Assumed*	*100 Hz*
F_{Z1}	$\dfrac{1}{2\pi \times (R1 + R3) \times C2}$	*Chosen 150 Hz*
F_{Z2}	$\dfrac{1}{2\pi \times R2 \times C1}$	*Chosen 850 Hz*
F_{P1}	$\dfrac{1}{2\pi \times R2 \times C3}$	*Chosen 10 kHz*
F_{P2}	$\dfrac{1}{2\pi \times R3 \times C2}$	*Chosen 75 kHz*

Running MPAC Analysis on the Boost Converter

To perform the MPAC analysis, the design must be tested for its transient response. A general-purpose voltage source is connected at the node where the perturbation of 0.001 V magnitude is injected as shown in Figure 3. With this configuration, we can obtain the Bode plot of the overall open-loop transfer function of the DC-DC boost converter.

In this design example, an experiment is created to plot the Bode plots of the overall open-loop transfer function at different operating points. The experiment and the settings for PAC analysis are shown in Figure 6. PAC analysis is performed using the Multi-tone mode. The settings of the ramp, settle and acquire are increased to minimize the noise on the frequency analysis output and get smoother waveforms. Refer to the SaberRD online documentation for more details on these settings. Assigning much larger values leads to increased simulation times without much improvement in the simulation output. Observe that the output voltage (`vout`), feedback voltage (`vfb`), and control voltage (`vctrl`) are selected as the output signals. The simulation will have the frequency response waveforms of these three signals only. Dividing `vout` by `vfb` gives the Bode plot of the overall open-loop transfer function. If you divide the signal `vctrl` by `vfb`, you get the Bode plot of the compensator transfer function. Similarly, if you divide `vout` by `vctrl`, you get the Bode plot of the open-loop system transfer function. By measuring the gain and phase margins, we can assess the stability of the system from these Bode plots.

Figure 6: *Experiment and PAC analysis settings*

A MultiVary loop (`frequency_plots`) is used to set different operating conditions before performing the PAC analysis. We have the flexibility to vary multiple parameters for a given instance of PAC analysis without having to include nested vary loops. Shown in Figure 7 are the MultiVary parameters that include varying input DC voltage and load resistance simultaneously. Similarly, we can add multiple parameters to vary for each iteration of the PAC analysis. By using this method, we can obtain the frequency response of the system for different operating conditions automatically without editing each parameter manually, and run the analysis.

frequency_plots   ✕

Run: [   ]   Parameter Name: [   ]

Runs	:boost_cl_pac:v_dc.v_d…	:boost_cl_pac:r.rload:rn…
1	4.5	15
2	4.5	20
3	4.5	30
4	5.5	15
5	5.5	20
6	5.5	30

Figure 7: *MultiVary analysis parameter settings*

How to Run the Experiment

1) Open the design `boost_cl_pac.ai_dsn` from the attached design files.

2) Go to the Simulate Tab. Select **Experiment** in the Analysis and `performance_testing` in the experiments list.

3) Click the **GO** button.

4) Once the experiment progress is 100%, the results pane shows the graphs "`bode_plot`" and "`phase_margin`".

5) Double-click graph "`bode_plot`" and the Bode plot of the overall open-loop transfer function shown in Figure 8 is displayed. Similarly, double-clicking the graph "`phase_margin`" shows the phase margin measurement for the different operating conditions simulated, as shown in Figure 9.

Simulation Results and Observations

The closed-loop DC-DC boost converter design is simulated for different operating conditions using MultiVary and Experiment Analyzer. Figure 8 shows the Bode plot of the overall open-loop transfer function.

The difference between the curves with varying load is minimal because of the insignificant series path impedance. But when the input DC voltage is varied between the two limits of the specified range (Table 1), the resonant tank frequency is changing. This can be seen in Figure 8, where the peak in the magnitude plot is moving from 900 Hz to 1.2 kHz. The change in input DC voltage causes a shift in switching duty cycle, thereby moving the LC resonant tank frequency in a boost converter.
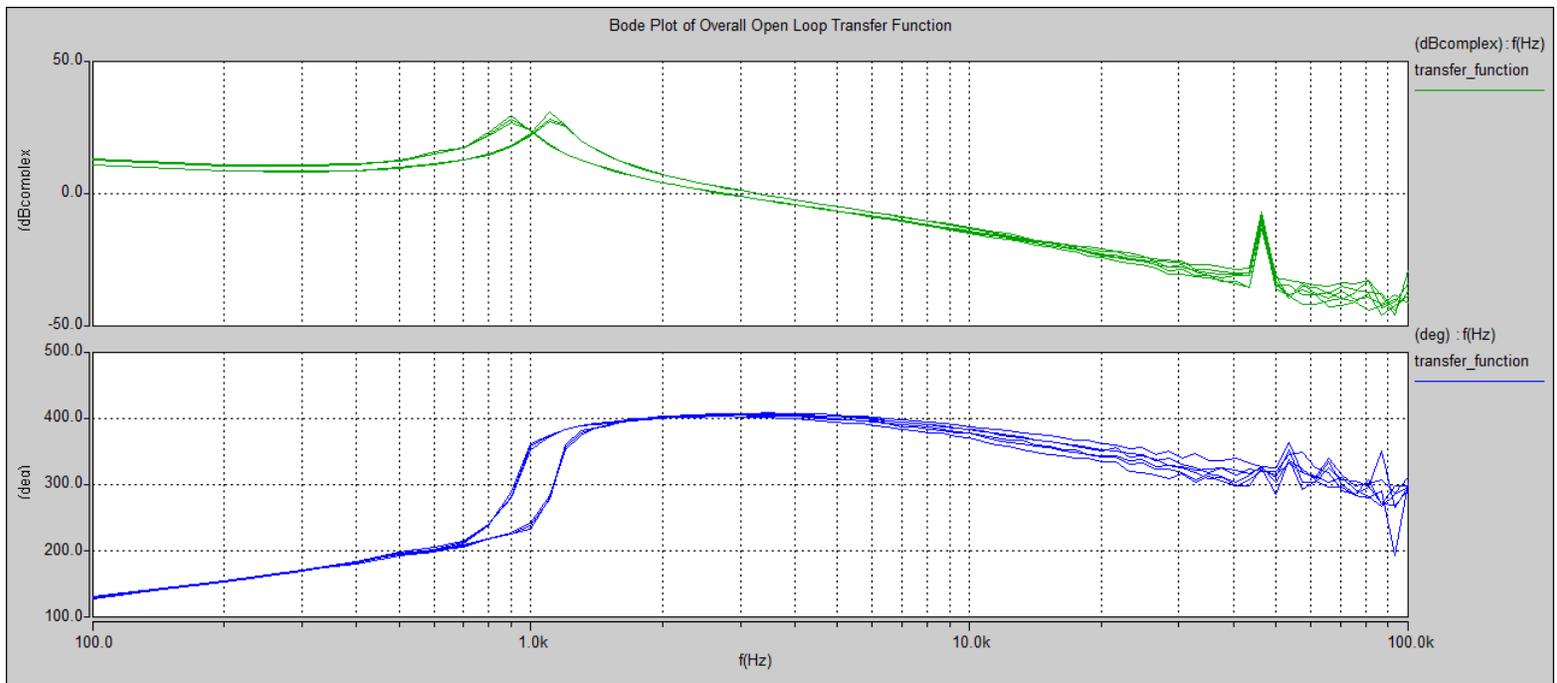
Figure 8: *Bode plots of the overall open-loop transfer function*

Figure 9 is a plot of the measurement of phase margin of all the Bode plots that represent the simulated operating points. The phase measurement is rounded off at 360^0 so as to get the measurement within the maximum of 360^0. It can be seen in the figure that the phase margin for the first operating condition/iteration is less than 45^0. But for stable operation, the phase margin should always be greater than 45^0.

Figure 9: *Phase margin measurement of the overall open-loop transfer function*

To ensure stability over the entire operating range of the converter, we need to look at the first iteration. From Figure 7, the first operating condition is where the input DC voltage is 4.5 V, and the load is 15 Ω. This operating condition is the worst case condition where the input voltage is at its lowest value, and the load is at its highest value. If the converter were designed and tested for just the nominal case,

stability issues would show up when the converter operates in these extreme conditions. So, evaluating the design in all possible operating conditions helps to uncover as many design issues as possible. Many design issues can be rectified by performing detailed analysis under all operating conditions and the number of hardware iterations can be minimized.

Design Change and Verification

The compensator is the main subsystem that maintains the converter in a stable state. So, we will have to revisit the type-3 compensator design. In the present case, we have to increase the phase boost by around 2 degrees. For this, we have to increase the distance between the second zero frequency (F_{Z2}) and the first pole frequency (F_{P1}). From Table 3, we can see that the frequency, F_{P1} can be changed from 10 kHz to 20 kHz by reducing the capacitance value of C_3 to half of its current value.

To check the effectiveness of this design change, there is another experiment available in the design.

How to Run the Experiment

1) Go to Simulate Tab. Select "Experiment" in the Analysis and `pm_improvement` in the experiments list.

2) Click the GO Button.

3) Once the experiment progress is 100%, the results pane shows the graph "`bode_plot`".

4) Double click on graph "`bode_plot`" and the Bode plot of the overall open-loop transfer function shown in Figure 10 is displayed.

Simulation Results and Observations

The experiment alters the value of capacitance C3 from 322 pF to 161 pF in the compensator design and runs a PAC analysis in Multi-tone mode. The resultant Bode plot of the overall open-loop transfer function is shown in Figure 10.
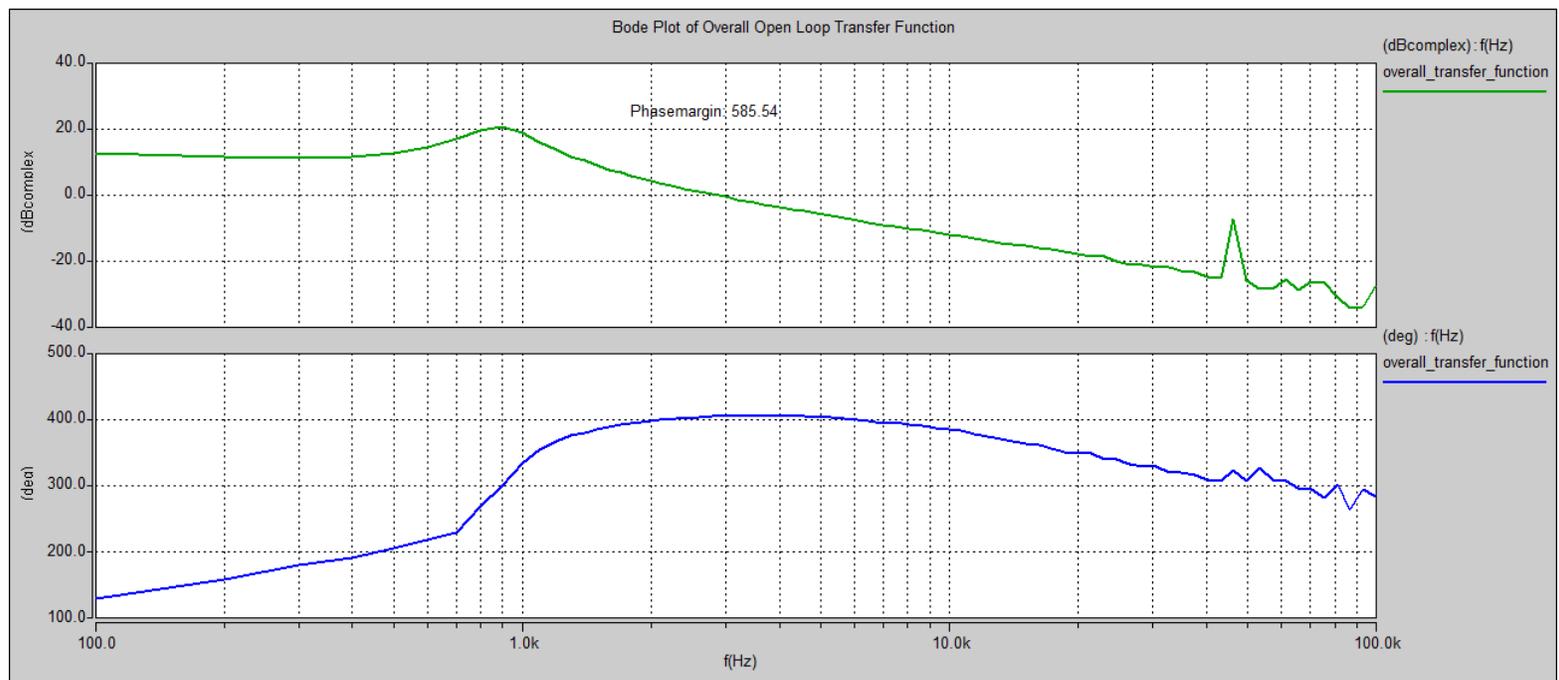

Figure 10: *Bode plots of the overall open-loop transfer function with design change*

The phase margin measurement shown in Figure 10 is with respect to -180^0. To calculate the actual phase margin, we subtract 180^0 and 360^0. This value is $585.5-180-360 = 45.5^0$. This phase margin satisfies the requirement for stability, and we can see from Figure 9 that the phase margin for other operating conditions will be more than 45^0.

Through these experiments, we assessed the loop stability for the entire range of input DC voltage and load current. Using this automated approach, the system can be thoroughly tested for other parameter changes also.

Conclusions

The frequency analysis of a closed-loop DC-DC boost converter with voltage feedback and type 3 compensator is performed using Multi-tone PAC analysis. The PAC analysis is helpful to understand the frequency response (Bode plot) of the switching circuit without having to derive an average model for it. By using MultiVary and testing all possible combinations of the operating conditions, design issues can be uncovered and rectified through simulation. This reduces the number of hardware iterations, thereby saving cost and time.

References

[1] http://www.ti.com/lit/an/slva633/slva633.pdf